Empirical Verification of Recently Proposed Hadron Mass Formulas

Eckehard W. Mielke
International Centre for Theoretical Physics, Trieste, Italy

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For the nucleon resonances as well as for the heavy vector mesons different mass formulas are compared. Those corresponding to a band of rotational excitations of the particle seem to agree rather well with the existing data.

It has become an increasing challenge to relate the overwhelming flow of data [1] on newly discovered particles to theoretical models. As a rather humble step towards this formidable goal it seems worthwhile to point out any regularities in the particle spectrum. Since this is particularly necessary for the corresponding energy levels we shall deal with the following issues:

A) The angular momentum dependence of several deviating mass formulas will be compared with the data on the rotational bands of *nucleon resonances*.

B) Three mass formulae for the recently discovered "photon family" of isospin singlet mesons are proposed and then related to the experiments. As a result, the mass of the "toponium" may be predicted.

C) As a concluding remark some speculative observations on the origin of Barut's lepton mass formula [2] will be presented.

A) Nucleon Resonances

Stimulated by a recent work of MacGregor [3] the band of nucleon and Δ-resonances will be considered. For these systems, the observed correlation between higher spin values and increasing energies suggests a dynamics corresponding to a rotational, "internal" motion of the particle.

Such a phenomenon has been demonstrated in an *exactly* solvable "toy" model of non-linear and non-local interacting Klein-Gordon fields [4]. For a certain non-trivial stationary ansatz φ_0 (see Eq. (6) of Ref. [5]) the non-locality may be omitted. Then the field energy of the obtained soliton-type solutions yields a mass formula of the form [5]

$$M = m_0 + a L(L+1)\,, \tag{1}$$

where L denotes the quantum number of the angular momentum of the soliton and m_0 and a are arbitrary constants with the dimension of a mass. Although resembling the Gürsey-Radicati formula [6] common to non-relativistic theories with internal SU(6) symmetry, it should be noted that our result (1) is the outcome of a Lorentz invariant model.

In quite a different context, the classical black soliton type [7] solutions of the coupled Einstein-Yang-Mills equation have been analysed. Arguments have been advanced [8] that such Kerr-Newman type objects obey the mass relation [9]

$$M^2/M^{*2} = \left\{1 - \beta Y + \frac{\alpha}{4}\left[I(I+1) - \frac{1}{4}Y^2\right]\right\}^2 + \frac{1}{4}J(J+1) \tag{2}$$

of the Gell-Mann-Okubo type. Generalizing Wheeler's "no hair" conjecture, the mass of the black soliton depends only on the quantum numbers I of the isospin, Y of the hypercharge and J of the total angular momentum but not on other details of the internal structure ($\alpha = g^2/\hbar c$ is the dimensionless gauge coupling constant and β a phenomenological constant resulting from a symmetry breaking). In the case of nucleon resonances we may simply probe the black soliton mass formula (2) in the simplified form

$$M^2 = m_0^2 + a J(J+1)\,. \tag{3}$$

Mass formulas of this kind have already a long history. If a "relativistic rotator" is considered in the context of dynamical groups the relation (3) results from the eigenvalues of the first Casimir operator of an irreducible continuous representation of the de Sitter group SO(1,4) [10]. This

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formula has been found to agree rather well with part of the meson [11, 12] as well as with the baryon [13, 14] spectrum. For both kinds of particles the fit to the data can be made with a residual moment of inertia squared $a = 0.28\,(\text{GeV})^2$ which is universal and rather close to the value $\sim \frac{1}{4}\,(\text{GeV})^2$ suggested by the generally relativistic formula (2).

This and other reasons have encouraged attempts [10, 15, 16] to interpret hadrons as micro-de Sitter universes. It has been speculated [8, 9, 17] that such a topology of the small-scale structure of space-time may also provide a solution to the problem of quark confinement.

In any case (2) and (3) belong to the few mass formulas which can be deduced from relativistic theories which are also in accordance with Einstein's principle of general covariance. Furthermore, the left-hand side of (2) and (3) really is the proper rest mass squared according to fundamental principles of generally covariant field theories.

In Table 1 we compare (1), (3) and the relation

$$M^2 = m_0^2 + aJ \tag{4}$$

for linearly rising Regge trajectories with the data taken from [1] and [18]. Following MacGregor [19] the assignment

$$L = J + \tfrac{1}{2} \tag{5}$$

common in nuclear physics has been used in (1). The fit has been made for higher spin values where the J dependence is expected to dominate over other nuclear effects.

We have found that the rotational relation (1) agrees rather well, the "black soliton" formula (3) fairly well and the Regge type formula (4) only poorly with experiments (see Tables 1 and 2). According to (1) the $\mathcal{N}(3030)$ and $\mathcal{N}(3690)$ resonances should be identified as $J^P = 13^+/2$ and $J^P = 15^-/2$ states, respectively. However a $J^P = 7^-/2$ state is missing at 1940 MeV. The state $\mathcal{N}(1360)$ of Ref. [18] has been included in Table 1 following the suggestion of MacGregor [20], although it is questionable if it can be regarded as a proper nucleon resonance.

B) Heavy Mesons

Recently considerable attention has been drawn to the "photon family" of hadronic mesons, i.e. to those vector mesons ϱ, ω, φ, ψ/J and possibly $\mathcal{S}$ which carry the quantum numbers of the photon.

Using a generalization of the canonical commutation relations of nonrelativistic quantum mechanics Saavedra and Utreras [21] have deduced a geometric

Table 1. Single nucleon rotational bands.

Nucleon resonances	J^P	M_{exp} (MeV/c^2)	$M \sim L(L+1)$	$\sqrt{M^2 \sim J(J+1)}$	M^2_{exp} (GeV^2/c^4)	$M^2 \sim J(J+1)$	$M^2 \sim J$
$\mathcal{N}(1360)$?	$-\frac{1^-}{2}$	~ 1360	1340	1324	~ 1.85	1.75	0.09
$\mathcal{N}(1470)$	$\frac{1^+}{2}$	1400–1480	1400	1375	2.16 ± 0.29	1.89	1.20
$\mathcal{N}(1520)$	$\frac{3^-}{2}$	1510–1530	1520	1520	2.31 ± 0.19	2.31	2.31
$\mathcal{N}(1688)$	$\frac{5^+}{2}$	1670–1690	1700	1734	2.85 ± 0.22	3.01	3.42
?	$\frac{7^-}{2}$		1940	1995		3.98	4.54
$\mathcal{N}(2220)$	$\frac{9^+}{2}$	2150–2300	2240	2287	4.93 ± 0.67	5.23	5.65
$\mathcal{N}(2600)$	$\frac{11^-}{2}$	2580–2700	2600	2600	6.76 ± 1.04	6.76	6.76
$\mathcal{N}(3030)$	$\left(\frac{13^+}{2}\right)$	~ 3030	3020	2927	9.18 ± 1.21	8.57	7.87
$\mathcal{N}(3690)$?	$\left(\frac{15^-}{2}\right)$	~ 3690	3500	3264		10.65	8.99

Comparison of three different mass formulas all fitted for $\mathcal{N}(1520)$ and $\mathcal{N}(2600)$. Data taken from Refs. [1] and [12].

Table 2. Δ-nucleon states.

Δ Resonances	J^P	M_{exp} (MeV/c^2)	$M \sim L(L+1)$	$\sqrt{M^2 \sim J(J+1)}$	M^2_{exp} (GeV^2/c^4)	$M^2 \sim J(J+1)$	$M^2 \sim J$
Δ(1550) ?	$\frac{1^+}{2}$	$\sim$ 1550	1586	1581	$\sim$ 2.40	2.50	2.02
Δ(1650)	$\frac{1^-}{2}$	1600–1650			2.72 ± 0.23		
Δ(1670)	$\frac{3^-}{2}$	1630–1740	1670	1670	2.79 ± 0.33	2.79	2.79
Δ(1890)	$\frac{5^+}{2}$	1890–1930	1794	1808	3.57 ± 0.47	3.27	3.56
Δ(1950)	$\frac{7^+}{2}$	1910–1950	1962	1985	3.80 ± 0.47	3.94	4.32
Δ(2160)	?	2150–2280			4.67 ± 0.65		
Δ(2300) ?	$\frac{9^+}{2}$	$\sim$ 2217	2169	2192	$\sim$ 4.92	4.81	5.09
Δ(2420)	$\frac{11^+}{2}$	2380–2450	2420	2420	5.86 ± 0.73	5.86	5.86
Δ(2750) ?	$\frac{13^-}{2}$	2650–2794	2711	2666	$\sim$ 7.56	7.11	6.63
Δ(2950) ?	$\frac{15^+}{2}$	2850–2990	3045	2923	$\sim$ 8.70	8.55	7.40

Comparison of three different mass formulas all fitted for Δ(1670) and Δ(2420). Data taken from Ref. [1].

progression type mass formula

$$M_f = m\,3^{f-3}, \quad f \geqq 3, \tag{6}$$

for a high energy system. This relation reproduces reasonably well the masses of those heavy photons which according to current philosophy have a pure $(q_f \bar{q}_f)$ quark content.

However, guided by the success of the rotational formula (1) in the case of nucleon resonances, the related formula

$$M_f = m_0 + \frac{3}{2}\pi^2 \frac{m_e}{\alpha} \sum_{\iota=0}^{f-3} \iota(\iota+1) = m_0 + 3\pi^2 \frac{m_e}{\alpha} \binom{f-1}{3}, \quad f \geqq 3 \tag{7}$$

is proposed here. According to it the energy of a heavy photon appears to be built up from an entire band of internal rotational excitations. (The thresholds in e^+e^- collisions are suspected by Lehmann [22] to obey a similar law.)

A further ad hoc mass relation may read

$$M_f = m_0 + \frac{3}{2}\pi^2 \frac{m_e}{\alpha} \sum_{k=1}^{f-2} k!, \quad f \geqq 3. \tag{8}$$

Table 3 shows that our phenomenological ansatz (7) is in good agreement with the ground states of the heavy photons. More important, it predicts the mass of "toponium" to be at $M_6 = 21.8$ GeV/c^2, whereas (6) and (8) would lead to higher values of 27.5 GeV/c^2 and 34.2 GeV/c^2, respectively. The second value agrees with the expectations of "every third expert" (see Table 2.1 of a review on "quarkonia" by Krammer and Krasemann [22]).

Mesons	M (MeV/c^2)		Theoretical		
$I^G(J^P)C_n = 0^-(1^-)_-$	nominal	exp.	$M_f \sim \binom{f-1}{3}$	$M_f \sim 3^{f-3}$	$M_f \sim \sum_{k=1}^{f-2} k!$
φ ($s\bar{s}$)	1020	1019.6 ± 0.1	1019.6	1019.6	1019.6
J/ψ ($c\bar{c}$)	3100	3097 ± 1	3093	3059	3093
$\mathscr{S}$ ($b\bar{b}$)	9460	9458 ± 6	9313	9176	9313
($t\bar{t}$) ?			21753	27529	34194

Table 3. Heavy photons. The suggested mass formulas are fitted to the data [1] for the first state only.

According to earlier measurements [23] a new quarkonium family based on a further charge $\frac{2}{3}$ quark is unlikely to be found below an energy of 16.5 GeV. In a recent search for a new quark flavour [24], no bound states lying below the continuum threshold have been observed in the centre-of-mass region 29.8 to 31.8 GeV/c^2. More recently, however, three events with energies larger than 20 GeV have been observed at CERN in an experiment [25] measuring prompt muon production.

C) Lepton Mass Formula

Our guess of the heavy photon mass relation (7) has been considerably influenced by the mathematical structure of Barut's lepton mass formula [2] (compare also with Rosen's [26])

$$m_l/m_e = 1 + \frac{3}{2\alpha} \sum_{k=1}^{l} k^4 \tag{9}$$

which agrees extremely well with observations. By employing formula (0.121.4) of Gradshteyn and Ryzhik [27] (9) can be given the equivalent form

$$m_l/m_e = 1 + \frac{1}{20\alpha} \cdot (2l+1)\,l(l+1)[3l(l+1)-1]. \tag{10}$$

For the μ meson the relation (9) can be derived on the basis of a magnetic self-interaction of the electron [28]. In the general case we may remind ourselves of the identity

$$\sum_{m=-l}^{l} Y_l^m(\theta,\varphi)^* \, Y_l^m(\theta,\varphi) = (2l+1)/4\pi \tag{11}$$

for spherical harmonics which allowed us to obtain a certain non-trivial exact solution φ_0 in a non-linear $|\varphi|^6$ model [5]. Comparing this with (10) one may be tempted to speculate that non-linear and non-local self-interactions such as those proposed by Drechsler [29] could play a pivotal role for our understanding of the "internal" structure of leptons.

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